

Universality of non-equilibrium dynamics of CFTs from holography

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Motivated by a low-energy effective description of gauge theory/string theory duality, we conjecture that the dynamics of $SO(4)$ -invariant states in a large class of four-dimensional conformal gauge theories on S^3 with non-equal central charges $c \neq a$ are universal on time scales $t_{\text{universal}} \propto c (\mathcal{E} - \mathcal{E}_{\text{vacuum}})^{-1}$, in the limit where the energy $\mathcal{E} \rightarrow \mathcal{E}_{\text{vacuum}}$. We show that low-energy excitations in $c \neq a$ CFTs do not thermalize in this limit. The holographic universality conjecture then implies that within the Einstein-scalar field system (dual to theories with $c = a$), AdS_5 is stable to spherically symmetric perturbations against formation of trapped surfaces within time scales $t_{\text{universal}}$.

Holographic universality conjecture.— Over the years the holographic gauge/gravity correspondence has developed into a powerful tool to probe the physics of strongly interacting quantum systems by mapping them into problems in classical gravity [1]. The quintessential example is the correspondence between supersymmetric $\mathcal{N} = 4$ $SU(N)$ Yang-Mills theory and type IIB string theory in $AdS_5 \times S_5$. In the planar limit, $N \rightarrow \infty$ and $g_{\text{YM}}^2 \rightarrow 0$ with $g_{\text{YM}}^2 N$ kept fixed, quantum corrections on the string theory side can be neglected. Furthermore, for large 't Hooft coupling, $g_{\text{YM}}^2 N \rightarrow \infty$, the holographic dual is captured by classical Einstein gravity in a five-dimensional AdS spacetime. It is well understood that successive $\mathcal{O}((g_{\text{YM}}^2 N)^{-3/2})$ 't Hooft coupling corrections translate into higher derivative $\mathcal{O}((\alpha')^3)$ corrections, while non-planar $1/N$ corrections correspond to g_s string loop corrections.

Other examples of holographic correspondence for four-dimensional superconformal gauge theories involve $AdS_5 \times X_5$ string theory, where X_5 is a general Sasaki-Einstein manifold [2]. Once again, a Kaluza-Klein reduction on X_5 (in the planar limit and for large 't Hooft coupling) produces a sector of classical Einstein gravity in AdS_5 ,

$$S = \frac{1}{2\ell_p^3} \int_{\mathcal{M}_5} d^5\xi \sqrt{-g} \left(\frac{12}{L^2} + R + \mathcal{L}_{\text{matter}} \right), \quad (1)$$

where $\mathcal{L}_{\text{matter}}$ is a Lagrangian for the gravitational bulk matter sector which encodes the spectrum of operators of the dual CFT with small anomalous dimensions.

A common feature of strongly coupled gauge theories with gravity dual (1) is the equality between the two central charges c and a parameterizing the conformal anomaly of a four-dimensional CFT in a curved space-time \mathcal{M}_4 [3]. More generally, the conformal anomaly takes the form

$$\begin{aligned} \langle T^\mu{}_\mu \rangle_{\text{CFT}} &= \frac{c}{16\pi^2} I_4 - \frac{a}{16\pi^2} E_4, \\ E_4 &= r_{\mu\nu\rho\lambda} r^{\mu\nu\rho\lambda} - 4r_{\mu\nu} r^{\mu\nu} + r^2, \\ I_4 &= r_{\mu\nu\rho\lambda} r^{\mu\nu\rho\lambda} - 2r_{\mu\nu} r^{\mu\nu} + \frac{1}{3} r^2, \end{aligned} \quad (2)$$

where E_4 and I_4 correspond to the four-dimensional Euler density and the square of the Weyl curvature of $\mathcal{M}_4 = \partial\mathcal{M}_5$. However, for the action (1), both central charges reduce to [4]

$$c = a = \frac{\pi^2 L^3}{\ell_p^3}, \quad (3)$$

where L is the asymptotic AdS_5 radius of curvature, and ℓ_p is a five-dimensional Planck length.

Superconformal gauge theories with $c \neq a$ can also be described in a holographic framework, by supplementing the effective action (1) with a five-dimensional Gauss-Bonnet (GB) term [5],

$$S = \frac{1}{2\ell_p^3} \int_{\mathcal{M}_5} d^5z \sqrt{-g} \left(\frac{12}{L^2} + R + \mathcal{L}_{\text{matter}} + \frac{\lambda_{\text{GB}}}{2} L^2 (R^2 - 4R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}) \right), \quad (4)$$

with the identifications (causality constrains $-\frac{7}{36} \leq \lambda_{\text{GB}} \leq \frac{9}{100}$ [6])

$$\begin{aligned} c &= \frac{\pi^2 \tilde{L}^3}{\ell_p^3} \left(1 - 2 \frac{\lambda_{\text{GB}}}{\beta^2} \right), & a &= \frac{\pi^2 \tilde{L}^3}{\ell_p^3} \left(1 - 6 \frac{\lambda_{\text{GB}}}{\beta^2} \right), \\ \tilde{L} &\equiv \beta L, & \beta^2 &\equiv \frac{1}{2} + \frac{1}{2} \sqrt{1 - 4\lambda_{\text{GB}}}. \end{aligned} \quad (5)$$

The vacuum state of a dual CFT is described by the solution of the effective action (4) with the matter sector turned off,

$$|0\rangle_{\text{CFT}} \iff \mathcal{L}_{\text{matter}} = 0. \quad (6)$$

The dual to the vacuum state of a CFT on a three-sphere S^3 is then global AdS_5 ,

$$ds^2 = \frac{L^2 \beta^2}{\cos^2 x} (-dt^2 + dx^2 + \sin^2 x d\Omega_3^2), \quad (7)$$

where $d\Omega_3^2$ is the metric of S^3 . Following holographic renormalization of GB gravity developed in [5, 7], we find

that the vacuum energy (the mass) of (7), or the Casimir energy from the boundary CFT perspective, is

$$\mathcal{E}_{\text{vacuum}} = \frac{3a}{4\tilde{L}}. \quad (8)$$

The vacuum of a CFT is static. By contrast, a generic non-equilibrium state $|\xi\rangle$ of a CFT *evolves* with time. In a dual gravitational description this evolution corresponds to the dynamics of the coupled matter-gravity sector in (4), with the Dirichlet boundary conditions on the non-normalizable components of the excited bulk matter fields in $\mathcal{L}_{\text{matter}}$. Although

$$\frac{d}{dt}|\xi\rangle \neq 0,$$

the fact that a CFT represents a closed system implies that the energy of the state, \mathcal{E}_ξ , is conserved,

$$\frac{d}{dt}\mathcal{E}_\xi = 0. \quad (9)$$

We will argue in the following sections using the multiscale analysis of [10] that in the limit of small $(\mathcal{E}_\xi - \mathcal{E}_{\text{vacuum}})/\mathcal{E}_{\text{vacuum}}$, the dynamics of the bulk are well described by the leading order gravitational self-interaction of the matter up to a time $t_{\text{universal}} \propto c (\mathcal{E}_\xi - \mathcal{E}_{\text{vacuum}})^{-1}$, where c is the CFT central charge. It is therefore reasonable to assume that on this time scale, the physics of the strongly coupled dual CFT is also described by this approximation. Moreover, to leading order the gravitational self-interaction is independent of λ_{GB} . We are thus led to the following *holographic universality conjecture*:

Consider an $SO(4)$ invariant state $|\xi\rangle$ of a CFT. The dynamics of the CFTs are universal in the limit $(\mathcal{E}_\xi - \mathcal{E}_{\text{vacuum}})/\mathcal{E}_{\text{vacuum}} \rightarrow 0$: Apart from a simple rescaling, they are insensitive to the central charge difference $(c - a)/c$ up to time scales $t_{\text{universal}}$.

An important consequence of the above is the stability of AdS_5 on time scales $t_{\text{universal}}$:

CFTs with non-equal central charges $(c - a)/c \neq 0$, which allow for an effective holographic description (4), cannot thermalize unless

$$\mathcal{E}_\xi - \mathcal{E}_{\text{vacuum}} \geq \frac{3c}{\tilde{L}} \begin{cases} \frac{1-\beta^2}{2\beta^2-1}, & \text{if } \lambda_{\text{GB}} > 0, \\ (\beta^2 - 1)(2\beta^2 - 1), & \text{if } \lambda_{\text{GB}} < 0. \end{cases} \quad (10)$$

Thus, under the assumption of universality, spherically symmetric Einstein gravity plus matter in AdS_5 , which is dual to CFTs with $c = a$, cannot thermalize within time scales of order $t_{\text{universal}}$ —the instability [11] (in the

sense of [8]) can arise only on time scales longer than $t_{\text{universal}}$ (as indicated by the plots in [9]).

The conformal gauge theories discussed in this work are somewhat restrictive. First, they must have a holographic dual within the supergravity approximation of type IIB string theory. Second, the higher derivative gravitational corrections (required to produce the central charge difference, $c \neq a$) are assembled into the GB combination (this is necessary to ensure that all the bulk equations of motion are of the second order). Finally, we impose some technical restrictions on the possible form of $\mathcal{L}_{\text{matter}}$ in (4). It would be interesting to relax these constraints, and also to consider generalizing the discussion to holographic quantum theories with hyperscaling violation [12].

Evidence for the conjecture.— The following assumption is vital for the holographic universality conjecture:

The leading order self-gravitational interaction of $\mathcal{L}_{\text{matter}}$ in (4) correctly captures the physics of a holographically dual CFT at low energy.

The matter sector in (4) represented by $\mathcal{L}_{\text{matter}}$ can be very complicated. Since we are interested in non-equilibrium dynamics of $SO(4)$ invariant states $|\xi\rangle$ with \mathcal{E}_ξ close to the vacuum energy, we take the approximation

$$\mathcal{L}_{\text{matter}} = \sum_i (-3\partial_\mu\phi_i\partial^\mu\phi_i - 3m_i^2\phi_i^2) + \mathcal{O}(\phi^6), \quad (11)$$

where the summation runs over the set of bulk scalar fields $\{\phi_i\}$, dual to the spectrum of operators $\{\mathcal{O}_i\}$ of dimensions $\{\Delta_i\}$ in a boundary CFT,

$$\Delta_i(4 - \Delta_i) = -m_i^2\tilde{L}^2. \quad (12)$$

We study the evolution of a state in a CFT, and, as a result, we impose Dirichlet boundary conditions on non-normalizable coefficients of $\{\phi_i\}$. Note that we restricted possible potential terms in (11), *i.e.*, we assumed that the matter sector is well approximated by essentially free bulk scalars. Such an approximation implies that leading nonlinearities in the bulk dynamics come from the interactions through gravity (minimal coupling), rather than from non-linear interactions within the matter sector itself. Additionally, we assume that $\Delta_i > 2$.

To proceed, we write the 5-dimensional metric describing an asymptotically AdS spacetime with $SO(4)$ symmetry in the form

$$ds^2 = \frac{L^2\beta^2}{\cos^2 x} \left(-Ae^{-2\delta} dt^2 + \frac{dx^2}{A} + \sin^2 x d\Omega_3^2 \right), \quad (13)$$

where $A(x, t)$ and $\delta(x, t)$ are scalar functions. We also take the scalar fields to be functions of (t, x) only: $\phi_i = \phi_i(t, x)$.

With the above ansatz, we obtain the equations of motion,

$$0 = \square \phi_i + \Delta_i(\Delta_i - 4)\phi_i, \quad (14)$$

$$A_{,x} = \frac{1}{\cos x(\beta^2 \sin^2 x + 2\lambda_{\text{GB}}(\cos^2 x - A))} \left(2 \sin x(\beta^2(1 + \sin^2 x)(\beta^2 - A) - \beta^2(\beta^2 - 1) \cos^2 x - 2\lambda_{\text{GB}}A(\cos^2 x - A)) \right) \\ - \frac{\beta^2 \sin^3 x \cos x}{A(\beta^2 \sin^2 x + 2\lambda_{\text{GB}}(\cos^2 x - A))} \sum_i \left(e^{2\delta}(\partial_t \phi_i)^2 + A^2(\partial_x \phi_i)^2 + \frac{A}{\cos^2 x} \Delta_i(\Delta_i - 4) \phi_i^2 \right), \quad (15)$$

$$\delta_{,x} = -\frac{\beta^2 \sin^3 x \cos x}{A^2(\beta^2 \sin^2 x + 2\lambda_{\text{GB}}(\cos^2 x - A))} \sum_i (e^{2\delta}(\partial_t \phi_i)^2 + A^2(\partial_x \phi_i)^2), \quad (16)$$

where \square is computed with (13), together with one constraint equation,

$$A_{,t} + \frac{2\beta^2 \sin^3 x \cos x A}{\beta^2 \sin^2 x + 2\lambda_{\text{GB}}(\cos^2 x - A)} \sum_i \partial_t \phi_i \partial_x \phi_i = 0. \quad (17)$$

There is an additional second order equation, which is however redundant due to the $SO(4)$ symmetry. Notice that the AdS_5 solution (7) is recovered with

$$\phi_i = \delta = 0, \quad A = 1. \quad (18)$$

We are interested in smooth solutions of (14)–(16), subject to the following boundary conditions. At the

origin, regularity implies

$$\begin{aligned} \phi(t, x) &= \phi_0(t) + \mathcal{O}(x^2), \\ A(t, x) &= 1 + \mathcal{O}(x^2), \\ \delta(t, x) &= \delta_0(t) + \mathcal{O}(x^2). \end{aligned} \quad (19)$$

At the outer boundary $x = \pi/2$ we introduce $\rho \equiv \pi/2 - x$ so that

$$\begin{aligned} \phi_i(t, \rho) &= \rho^{\Delta_i} (\phi_{\Delta_i}(t) + \mathcal{O}(\rho^2)), \\ A(t, \rho) &= 1 - M\rho^4 + \mathcal{O}(\rho^6) + \mathcal{O}(\rho^{2 \min_i (\Delta_i)}), \\ \delta(t, \rho) &= 0 + \mathcal{O}(\rho^{2 \min_i (\Delta_i)}). \end{aligned} \quad (20)$$

The parameter M in (20) is related to the conserved energy \mathcal{E}_ξ . Indeed, as described in [13] it is convenient to introduce the mass-aspect function $\mathcal{M}(t, x)$ as

$$A(t, x) = 1 - \frac{1}{2\lambda_{\text{GB}}} \left((2\lambda_{\text{GB}} - \beta^2) \sin^2 x + \left(4\lambda_{\text{GB}}(\beta^2 - 2\lambda_{\text{GB}})\mathcal{M}(t, x) \cos^4 x + (2\lambda_{\text{GB}} - \beta^2)^2 \cos^4 x - \beta^4(1 - 4\lambda_{\text{GB}}) \cos(2x) \right)^{1/2} \right). \quad (21)$$

Using (15) we conclude that

$$\mathcal{M}(t, x) = \frac{1}{2\beta^2 - 1} \int_0^x dz \frac{\tan^3 z}{A(t, z)} \sum_i \left[e^{2\delta}(\partial_t \phi_i)^2 + A^2(\partial_x \phi_i)^2 + \frac{A}{\cos^2 x} \Delta_i(\Delta_i - 4) \phi_i^2 \right]. \quad (22)$$

Furthermore,

$$M = \mathcal{M}(t, x) \Big|_{x=\frac{\pi}{2}}. \quad (23)$$

Using the machinery of holographic renormalization [5, 7], we compute the energy of $|\xi\rangle$

$$\mathcal{E}_\xi = \frac{3c}{4L\beta} \left(\frac{\beta^2 - 6\lambda_{\text{GB}}}{\beta^2 - 2\lambda_{\text{GB}}} + 4M \right) = \frac{3c}{4\tilde{L}} \left(\frac{a}{c} + 4M \right). \quad (24)$$

We will show now that to leading order in the backreaction, the dynamics of (14)–(16) is universal—apart from a simple rescaling, it is insensitive to λ_{GB} . We apply the “Two Time Framework” (TTF) introduced in [10] to this system: some number of possibly massive scalar fields coupled to GB gravity in AdS_5 . We account for the backreaction by introducing a parameter ϵ and the associated slow time

$$\tau \equiv s_1 \epsilon^2 t. \quad (25)$$

We expand the fields in terms of both the fast time t and slow time τ as

$$\begin{aligned}\phi_i &= \epsilon \left(\phi_{i,(1)}(t, \tau, x) + s_2 \epsilon^2 \phi_{i,(3)}(t, \tau, x) + \mathcal{O}(\epsilon^4) \right), \\ A &= 1 + s_2 \epsilon^2 A_{(2)}(t, \tau, x) + \mathcal{O}(\epsilon^4), \\ \delta &= s_2 \epsilon^2 \delta_{(2)}(t, \tau, x) + \mathcal{O}(\epsilon^4),\end{aligned}\quad (26)$$

where s_i are λ_{GB} -dependent constants. From (14) we find

$$\begin{aligned}\partial_t^2 \phi_{i,(1)} &= \phi_{i,(1)}'' + \frac{3}{\sin x \cos x} \phi_{i,(1)}' - \frac{\Delta_i(\Delta_i - 4)}{\cos^2 x} \phi_{i,(1)} \\ &\equiv -L_i \phi_{i,(1)}.\end{aligned}\quad (27)$$

The operator L_i has eigenvalues $\omega_{i,j}^2 = (2j + \Delta_i)^2$ ($j = 0, 1, 2, \dots$) and eigenvectors $e_{i,j}(x)$ (“oscillons”). Explicitly,

itly,

$$e_{i,j}(x) = d_{i,j} \cos^{\Delta_i} x {}_2F_1(-j, \Delta_i + j; 2; \sin^2 x), \quad (28)$$

with $d_{i,j}$ the normalization constants. The oscillons form an orthonormal basis under the inner product

$$(f, g) = \int_0^{\pi/2} f(x)g(x) \tan^3 x \, dx. \quad (29)$$

The general real solution to (27) is

$$\phi_{i,(1)}(t, \tau, x) = \sum_{j=0}^{\infty} (A_{i,j}(\tau) e^{-i\omega_{i,j}t} + \bar{A}_{i,j}(\tau) e^{i\omega_{i,j}t}) e_{i,j}(x), \quad (30)$$

where $A_{i,j}(\tau)$ are arbitrary functions of τ , to be determined later.

At $\mathcal{O}(\epsilon^2)$ the constraints (15)–(16) have solutions

$$A_{(2)}(x) = -\frac{1}{s_2(2\beta^2 - 1)} \frac{\cos^4 x}{\sin^2 x} \int_0^x \sum_i \left(|\partial_y \phi_{i,(1)}(y)|^2 + |\partial_t \phi_{i,(1)}(y)|^2 + \frac{\Delta_i(\Delta_i - 4)}{\cos^2 x} \phi_{i,(1)}(y)^2 \right) \tan^3 y \, dy, \quad (31)$$

$$\delta_{(2)}(x) = \frac{1}{s_2(2\beta^2 - 1)} \int_x^{\pi/2} \sum_i (|\partial_y \phi_{i,(1)}(y)|^2 + |\partial_t \phi_{i,(1)}(y)|^2) \sin y \cos y \, dy. \quad (32)$$

Note that we are using the gauge with $\delta_{(2)}(\pi/2) = 0$.

Finally, at $\mathcal{O}(\epsilon^3)$ we obtain the equations for $\phi_{i,(3)}$,

$$\partial_t^2 \phi_{i,(3)} + L_i \phi_{i,(3)} + \frac{2s_1}{s_2} \partial_t \partial_\tau \phi_{i,(1)} = S_{i,(3)}(t, \tau, x), \quad (33)$$

where the source term is

$$\begin{aligned}S_{i,(3)} &= \partial_t(A_{(2)} - \delta_{(2)}) \partial_t \phi_{i,(1)} - 2(A_{(2)} - \delta_{(2)}) L_i \phi_{i,(1)} \\ &\quad + (A_{(2)}' - \delta_{(2)}') \phi_{i,(1)}' + \frac{\Delta_i(\Delta_i - 4)A_{(2)}}{\cos^2 x} \phi_{i,(1)}.\end{aligned}\quad (34)$$

Note that by choosing

$$s_1 = s_2 = \frac{1}{2\beta^2 - 1} = \frac{1}{\sqrt{1 - 4\lambda_{\text{GB}}}}, \quad (35)$$

the dependence on λ_{GB} in TTF is completely factored out [14]. In other words, the rescaling (35) identifies the TTF equations for different λ_{GB} ,

$$\left(\epsilon^2, \lambda_{\text{GB}} \right) \iff \left(\epsilon_{\text{eff}}^2 = \frac{\epsilon^2}{\sqrt{1 - 4\lambda_{\text{GB}}}}, \lambda_{\text{GB}}^{\text{eff}} = 0 \right). \quad (36)$$

The identification (36) is the basis for our holographic universality conjecture: since the TTF framework is valid on slow time scales, we expect universality of low-energy, non-equilibrium dynamics of dual CFTs on time scales

$$t_{\text{universal}} \propto \epsilon^{-2} \propto c (\mathcal{E}_\xi - \mathcal{E}_{\text{vacuum}})^{-1}, \quad \epsilon \rightarrow 0. \quad (37)$$

It is important to emphasize that the the scalar field profile must be held fixed as the limit $\epsilon \rightarrow 0$ is taken. On the CFT side, this corresponds to fixing the state as the energy is taken to $\mathcal{E}_{\text{vacuum}}$. Were the profile allowed to vary, then backreaction could be increased by concentrating the field energy into a small region. Indeed, consider the initial condition prepared by a marginal operator with $\Delta_i = 4$, of the form

$$\begin{aligned}\phi_{(1)} \Big|_{t=0} &= 0, \\ \partial_t \phi_{(1)} \Big|_{t=0} &= \omega_j d_j \cos^4 x {}_2F_1(-j, 4 + j; 2; \sin^2 x),\end{aligned}\quad (38)$$

with $d_j = 2\sqrt{(j+1)(j+2)(j+3)}$. The mass parameter M corresponding to this profile is

$$M = \epsilon^2 \omega_j^2 = \epsilon^2 (4 + 2j)^2. \quad (39)$$

We can keep M fixed in the limit $\epsilon \rightarrow 0$ if we excite the oscillon with index $j \sim M^{-1} \gg 1$. Following (31) we estimate for $x \lesssim j^{-1}$

$$\begin{aligned}|A_{(2)}(x, t = \tau = 0)| &\sim \frac{\cos^4 x}{\sin^2 x} x^4 \omega_j^2 d_j^2 \\ &\sim \frac{M}{\epsilon^2} (x d_j)^2 \sim \frac{Mj}{\epsilon^2} (xj)^2.\end{aligned}\quad (40)$$

Thus, $|\epsilon^2 A_{(2)}|_{(t,\tau)=(0,0)}$ becomes of the same order as the leading contribution for $x \sim j^{-1}$ and the series expansion (26) is inconsistent. When we talk about the leading order backreaction in the limit $(\mathcal{E}_\xi - \mathcal{E}_{\text{vacuum}}) \rightarrow 0$, we always assume that $\epsilon \rightarrow 0$ with the initial profile shape kept fixed. A priori, this does not guarantee that during the evolution $|A_{(2)}(t, \tau)|$ will continue to remain bounded. We argue in the next section that if $|A_{(2)}(t, \tau)|$ is bounded initially, it must be bounded for all times.

Universal dynamics and AdS_5 (in)stability.— Consider $c \neq a$ CFTs with dual holographic descriptions ($\beta \neq 1$). Equilibrium thermal states of such models are described by the static solution

$$A = 1 - \frac{1}{2\lambda_{\text{GB}}} \left((2\lambda_{\text{GB}} - \beta^2) \sin^2 x + \left(4\lambda_{\text{GB}}(\beta^2 - 2\lambda_{\text{GB}})M \cos^4 x + (2\lambda_{\text{GB}} - \beta^2)^2 \cos^4 x - \beta^4(1 - 4\lambda_{\text{GB}}) \cos(2x) \right)^{1/2} \right),$$

$$\phi_i = 0, \quad \delta = 0. \quad (41)$$

It is straightforward to observe that (41) has a regular horizon only if

$$M \geq \begin{cases} \frac{1-\beta^2}{2\beta^2-1}, & \text{if } \lambda_{\text{GB}} > 0, \\ (\beta^2 - 1)(2\beta^2 - 1), & \text{if } \lambda_{\text{GB}} < 0. \end{cases} \quad (42)$$

Thus, generic non-stationary states in such CFTs cannot equilibrate in the limit $(\mathcal{E}_\xi - \mathcal{E}_{\text{vacuum}})/\mathcal{E}_{\text{vacuum}} \rightarrow 0$ [cf. eq. (10)].

Under the assumption of holographic universality (*i.e.*, that the leading order self-gravity dynamics in the bulk correctly capture the behavior of the CFT for $t < t_{\text{universal}} \propto \epsilon^{-2}$), these arguments imply that Einstein gravity in AdS_5 is stable within time scales $t_{\text{universal}}$ against scalar collapse of generic initial data with amplitude $\propto \epsilon$, in the limit $\epsilon \rightarrow 0$. Indeed, since black holes cannot form for arbitrarily small ϵ in Gauss-Bonnet gravity [15, 16], holographic universality implies TTF solutions cannot diverge in finite time. Thus, since TTF dynamics are independent of λ_{GB} , Einstein gravity in particular is stable for $t < t_{\text{universal}}$. Of course, there is no tension with conjectures for instability of AdS [17, 18], since collapse can still occur over longer time scales.

TTF validity within fully non-linear dynamics and String Theory.— The holographic universality conjecture (and the supporting evidence) assumes that the leading order self-gravitation faithfully captures the dynamics of a generic state $|\xi\rangle$ in the CFT, with a holographic dual, in the limit $(\mathcal{E}_\xi - \mathcal{E}_{\text{vacuum}})/\mathcal{E}_{\text{vacuum}} \rightarrow 0$. We stress that if this is not true, not only do the higher order terms in the perturbative expansion (26) become important, but so do the $\mathcal{O}(\phi^6)$ corrections to the scalar potential in (11).

In particular, if the higher order terms encoding the mass gap in GB gravity (42) are important, then the TTF equation for $A_{(2)}$ (31) might develop a singularity in a finite slow time $\tau \rightarrow \tau_{\text{singular}}$,

$$\lim_{\tau \rightarrow \tau_{\text{singular}}} A_{(2)} \rightarrow -\infty. \quad (43)$$

This would result in the formation of a trapped surface within time scales $\propto \epsilon^{-2}$; earlier than the scenario described in the previous section. It would be interesting to explore this further and to connect to numerical analysis in [13, 19]. However, numerical results to date (for $d = 4$, see figure in [9]) indicate that $t_{\text{collapse}} > t_{\text{universal}}$, consistent with the conjecture.

The universality conjecture presented in this Letter has been motivated in the holographic framework, within the supergravity approximation to String Theory. Thus, it relies on the validity of the effective gravitational action (4). Recently, it was pointed out that GB effective actions arising from a consistent theory of gravity must include a tower of massive higher-spin states [20]. These states can not be excited in $SO(4)$ -symmetric dynamics discussed here. It would be interesting to relax the symmetry constraint and re-analyze approach to equilibrium of the boundary CFT. Note, however, that from the gravitational perspective it is natural to expect black hole formation is most efficient in spherical settings.

While the initial conditions for the CFT dynamical evolution in the holographic framework are well described by the supergravity modes, the late-time dynamics, in particular the formation of small black holes, can require physics beyond the supergravity approximation. Specifically [21], the approach to equilibrium of the boundary CFT can proceed through the initial formation of the highly excited ball of strings in the bulk, which is expected to be entropically favourable compared to the corresponding Schwarzschild black hole [22]. Such a process necessitates decay of the supergravity modes into string states, which is unfortunately not well understood.

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